Appendices for “Heritability Estimation of Dichotomous Phenotypes Using a Liability Threshold Model on Ascertained Family-based Samples”

**Appendix (A)**

**Numerical analysis for optimization of the heritability in M-step of EM algorithm**

The first derivative of with respect to is given by

where . Then, the objective function becomes

Similarly, we can get the first derivative of with respect to as follows,

where and

Finally, is updated according to the following iterative steps using and which were calculated at the previous E-step,

**Appendix (B)**

**Numerical analysis for maximizing the global lower bound**

If we denote the global lower bound for the conditional log-likelihood as , then the first derivative of with respect to ,,is given by

Here, it should be noted that is equivalent to the equations (1) and (2) in the Method and Appendix (A). Using the chain rule, we can easily obtain the first derivative of with respect to as follows,

where is the probability density function for the standard normal. To apply Newton-Raphson algorithm for the objective function , we derive the first derivative of with respect to , , as follows,

and each term is given by

and

With these terms, we iteratively update using the following equation until convergence,